Conditional Probability Worksheet with Answers and Formula

1. Define conditional probability and provide the formula for it. Include conditions under which it is applicable.

2. Explain the difference between independent and dependent events with examples.

3. If a card is drawn from a standard deck of 52 cards, what is the probability that it is a king given that it is a face card?

4. A box contains 3 red, 5 blue, and 2 green balls. If a ball is drawn at random, find the probability that it is red given that it is not green.

5. State the Law of Total Probability. How is it applied in practical scenarios?

6. A die is rolled twice. What is the probability of getting a sum of 8 given that the first roll is a 5?

7. In a survey, 70% of people like coffee, and 40% like tea. 20% like both. What is the probability that a person likes coffee given that they like tea?

8. True or False: If $P(A|B) = P(A)$, events A and B are independent. Justify your answer.

9. A bag contains 4 black and 6 white marbles. Two marbles are drawn without replacement. What is the probability that the second marble is black given that the first marble is white?

10. Given the following table of student grades and attendance, calculate $P(A|B)$:

	Attended	*Absent*
Passed	30	10
Failed	5	5

11. Draw a Venn diagram showing two events A and B with their probabilities. Use the diagram to illustrate $P(A|B)$.

Solutions

1. Conditional probability is defined as the probability of event A occurring given that event B has occurred. The formula is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, where P(B) > 0.$$

2. Independent events are those where the occurrence of one event does not affect the occurrence of the other. For example, tossing a coin and rolling a die. Dependent events are those where one event affects the outcome of the other, such as drawing cards without replacement.

3. The probability of a king given it is a face card:

$$P(King|Facecard) = \frac{4}{12} = \frac{1}{3}.$$

4. The probability of red given not green:

$$P(Red|NotGreen) = \frac{3}{3+5} = \frac{3}{8}.$$

5. The Law of Total Probability states that:

$$P(A) = \sum P(A \cap B_i),$$

where $\{B_i\}$ is a partition of the sample space.

6. For the sum of 8 given the first roll is 5:

$$P(Sum = 8|Firstroll = 5) = \frac{1}{6}.$$

7. Probability of liking coffee given liking tea:

$$P(LikesCoffee|LikesTea) = \frac{0.2}{0.4} = 0.5.$$

8. True. If $P(A|B) = P(A)$, the events are independent because the probability of A is not affected by B.

9. Probability of second being black given the first is white:

$$P(Second = Black|First = White) = \frac{4}{9}.$$

10. From the table, $P(A|B)$:

$$P(A|B) = \frac{30}{30+5} = \frac{30}{35} = \frac{6}{7}.$$

11. For the Venn diagram, $P(A|B)$:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$